

Level 10 - Volume of 3D Shapes

Area and Volume Formulas

Circle	$A = \pi r^2$
Circle	$C = \pi d$ or $C = 2\pi r$
General Prisms	$V = Bh$
Cylinder	$V = \pi r^2 h$

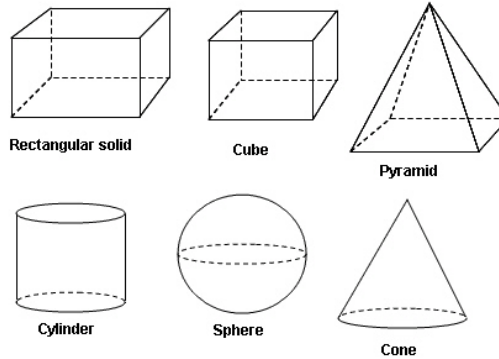
Sphere	$V = \frac{4}{3}\pi r^3$
Cone	$V = \frac{1}{3}\pi r^2 h$
Pyramid	$V = \frac{1}{3}Bh$

Key Fact

B = Area of the base!!!

3D Shapes We've Learned About

An object that has height, width and depth, like any object in the real world.



If an equilateral triangle is continuously rotated around one of its medians, which 3-dimensional object is generated?

- 1) cone
- 2) pyramid
- 3) prism
- 4) sphere

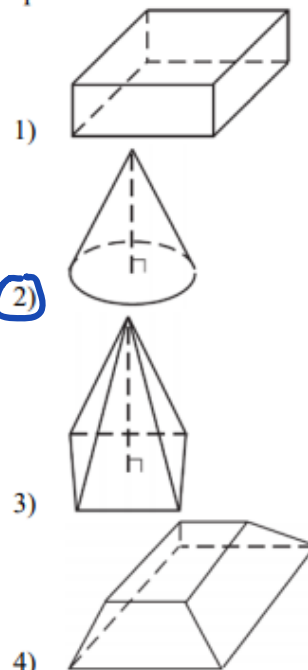
Which three-dimensional figure will result when a rectangle 6 inches long and 5 inches wide is continuously rotated about the longer side?

- 1) a rectangular prism with a length of 6 inches, width of 6 inches, and height of 5 inches
- 2) a rectangular prism with a length of 6 inches, width of 5 inches, and height of 5 inches
- 3) a cylinder with a radius of 5 inches and a height of 6 inches
- 4) a cylinder with a radius of 6 inches and a height of 5 inches

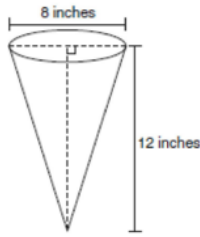
The cross section of a regular pyramid contains the altitude of the pyramid. The shape of this cross section is a

- 1) circle
- 2) square
- 3) triangle
- 4) rectangle

Which figure can have the same cross section as a sphere?



In the diagram below, a right circular cone has a diameter of 8 inches and a height of 12 inches.



$$V = \frac{1}{3} \pi r^2 \cdot h$$

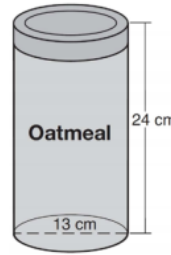
$$V = \frac{1}{3} \pi (4)^2 \cdot 12$$

$$V = 201$$

What is the volume of the cone to the nearest cubic inch?

- 1) 201
- 2) 481
- 3) 603
- 4) 804

Oatmeal is packaged in a cylindrical container, as shown in the diagram below.



$$V = \pi r^2 \cdot h$$

$$V = \pi (6.5)^2 \cdot 24$$

$$V = 1014\pi \text{ cm}^3$$

The diameter of the container is 13 centimeters and its height is 24 centimeters. Determine, in terms of π , the volume of the cylinder, in cubic centimeters.

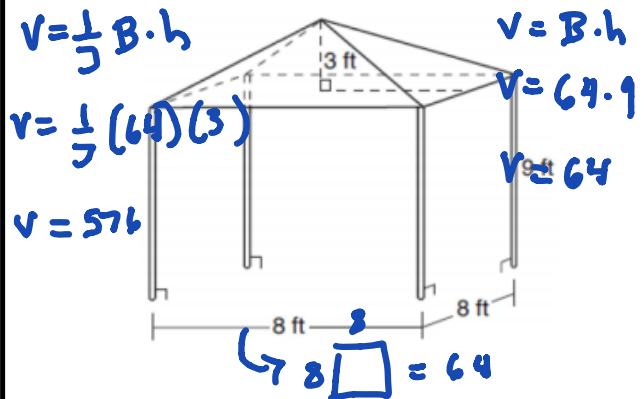
The diameter of a sphere is 15 inches. What is the volume of the sphere, to the nearest tenth of a cubic inch?

- 1) 706.9
- 2) 1767.1
- 3) 2827.4
- 4) 14,137.2

$$V = \frac{4}{3} \pi r^3$$

$$V = \frac{4}{3} \pi (7.5)^3$$

$$V = 1767.1 \text{ in}^3$$



$$V = \frac{1}{3} B \cdot h$$

$$V = \frac{1}{3} (64) (3)$$

$$V = 576$$

$$V = B \cdot h$$

$$V = 64 \cdot 9$$

$$V = 64$$

$$8 \times 8 = 64$$

What is the volume, in cubic feet, of space the tent occupies?

- 1) 256
- 2) 640
- 3) 672
- 4) 768

A cone shaped tank is filled with sand has a radius of 2.5 feet and a height of 9 feet. If sand weighs 90 pounds per cubic foot, what is the total weight of the sand in the full tank, to the nearest tenth.

$$V = \frac{1}{3} \pi r^2 \cdot h$$

$$V = \frac{1}{3} \pi (2.5)^2 \cdot 9$$

$$V = 58.90486225$$

$$W = V \cdot D$$

$$W = 58.90486225 \cdot 90$$

$$W = 5301.437625$$

An ice cream waffle cone can be modeled by a right circular cone with a base diameter of 6.6 centimeters and a volume of 54.45π cubic centimeters. What is the number of centimeters in the height of the waffle cone?

- 1) $3\frac{3}{4}$
- 2) 5
- 3) 15
- 4) $24\frac{3}{4}$

$$V = \frac{1}{3} \pi r^2 \cdot h$$

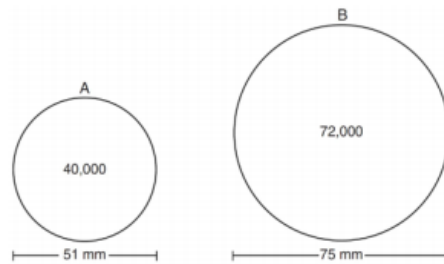
$$54.45\pi = \frac{1}{3} \pi (3.3)^2 \cdot h$$

$$[54.45 = \frac{1}{3} (3.3)^2 \cdot h]$$

$$\frac{163.35}{10.8} = \frac{10.89 \cdot h}{10.89}$$

$$15 = h$$

During an experiment, the same type of bacteria is grown in two petri dishes. Petri dish A has a diameter of 51 mm and has approximately 40,000 bacteria after 1 hour. Petri dish B has a diameter of 75 mm and has approximately 72,000 bacteria after 1 hour.



Determine and state which petri dish has the greater population density of bacteria at the end of the first hour.

$$D = \frac{\text{Pop}}{\text{Area}}$$

$$D = \frac{\text{Pop}}{\text{Area}}$$

$$D = \frac{40,000}{25.5^2 \cdot \pi}$$

$$D = \frac{72,000}{37.5^2 \cdot \pi}$$

$$D = 19.6 \text{ bacteria per } m^2$$

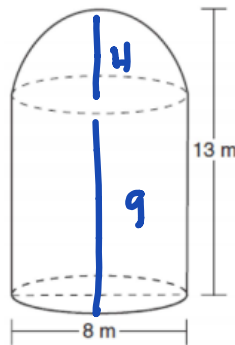
$$D = 16.3 \text{ bacteria per } m^2$$

A storage tank is in the shape of a cylinder with a hemisphere on the top. The highest point on the inside of the storage tank is 13 meters above the floor of the storage tank, and the diameter inside the cylinder is 8 meters. Determine and state, to the nearest cubic meter, the total volume inside the storage tank.

$$V = \frac{4}{3} \pi r^3$$

$$V = \frac{4}{3} \pi (4)^3$$

$$V = \frac{268.0825731}{2}$$



$$V = \pi r^2 \cdot h$$

$$V = \pi (4)^2 \cdot 9$$

$$V = 452.3893421$$

$$V = 134.0412866$$

$$V = 586 \text{ m}^3$$

Walter wants to make 100 candles in the shape of a cone for his new candle business. The mold shown below will be used to make the candles. Each mold will have a height of 8 inches and a diameter of 3 inches. To the *nearest cubic inch*, what will be the total volume of 100 candles?

$$V = \frac{1}{3} \pi r^2 \cdot h$$

$$V = \frac{1}{3} \pi (1.5)^2 \cdot 8$$

$$V = 18.84955592$$



Walter goes to a hobby store to buy the wax for his candles. The wax costs \$0.10 per ounce. If the weight of the wax is 0.52 ounce per cubic inch, how much will it cost Walter to buy the wax for 100 candles?

$$W = V \cdot D \quad W = 18.84955592 \times 0.52$$

$$C = W \cdot \text{price} \quad W = 9.801769079$$

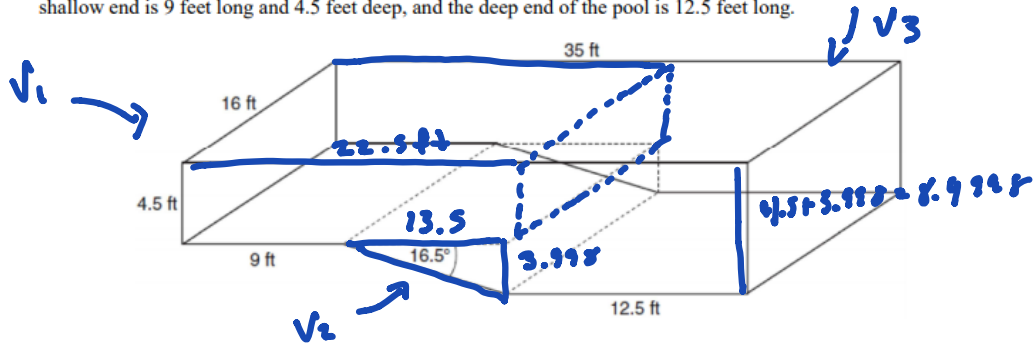
$$C = 9.801769079 \times 0.1$$

$$C = 0.9801769079$$

$$\times 100 \text{ candles}$$

$$\boxed{C = 98.02}$$

A rectangular in-ground pool is modeled by the prism below. The inside of the pool is 16 feet wide and 35 feet long. The pool has a shallow end and a deep end, with a sloped floor connecting the two ends. Without water, the shallow end is 9 feet long and 4.5 feet deep, and the deep end of the pool is 12.5 feet long.

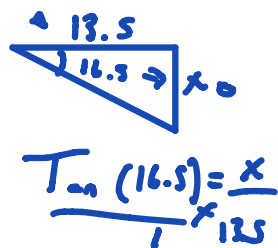


If the sloped floor has an angle of depression of 16.5 degrees, what is the depth of the pool at the deep end, to the *nearest tenth of a foot*? Find the volume of the inside of the pool to the *nearest cubic foot*. A garden hose is used to fill the pool. Water comes out of the hose at a rate of 10.5 gallons per minute. How much time, to the *nearest hour*, will it take to fill the pool 6 inches from the top? [1 ft³ = 7.48 gallons]

$$V_1 = B \cdot h \quad V_2 = B \cdot h$$

$$V_1 = (22.5 \times 16)(4.5) \quad V_2 = (12.5 \times 16)(8.9998)$$

$$V_1 = 1620 \quad V_2 = 1699.76$$



$$x = 13.5 \cdot \tan(16.5)$$

$$x = 3.998882182$$

$$V_3 = B \cdot h$$

$$V_3 = (\frac{1}{2} \times 13.5 \times 3.998) \times 16$$

$$V_3 = 431.784$$

$$\boxed{T_{total} = 3752 \text{ ft}^3}$$

$$V_{air} = 16 \times 35 \times .5$$

$$V_{air} = 280$$

$$3472 \text{ ft}^3$$

$$\begin{array}{r} \times \quad 7.98 \\ \hline 25970.56 \end{array}$$

$$\begin{array}{r} 3752 \\ - 280 \\ \hline 3472 \end{array}$$

← Total volume
of water

$$\frac{25970.56}{16.5} = 2473.38667$$

Gallons
Per minute

$$\frac{2473.38667}{60}$$

$$= \boxed{41 \text{ hours}}$$

Minutes