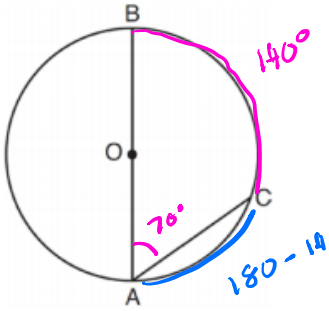
# Level 9 Study Guide

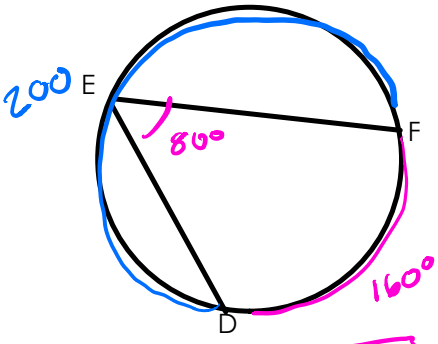
As shown in the diagram below, $\overline{AB}$ is a diameter of circle $O$, and chord $\overline{AC}$ is drawn.




If $m\angle BAC = 70$, then $m\overset{\frown}{AC}$ is

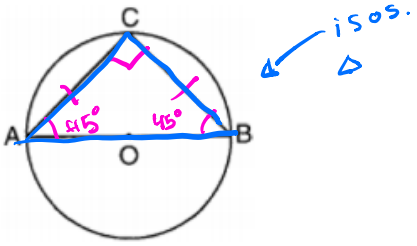
$m\overline{AC} = 40°$

In the circle below, $m\overset{\frown}{DEF} = 200°$. Find the $m\angle DEF$.




$\angle DEF = 80°$

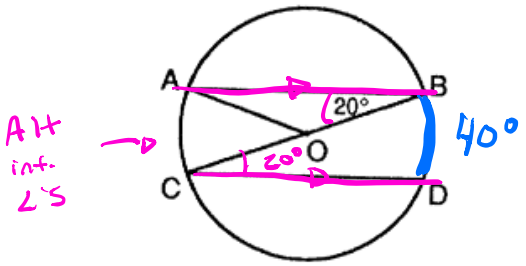
In the accompanying diagram, isosceles triangle $ABC$ is inscribed in circle $O$ with diameter $\overline{AOB}$. Find $m\angle CAB$.




$180 - 90 = \frac{90}{2} = 45°$

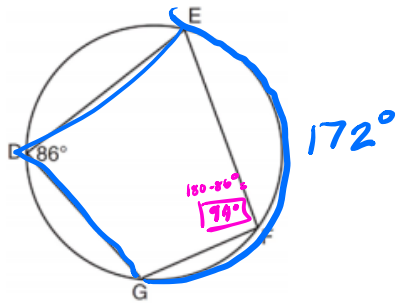
$\angle CAB = 40°$

In the accompanying diagram of circle $O$, $\overline{AB} \parallel \overline{CD}$, $\overline{BC}$ is a diameter, and radius $\overline{AO}$ is drawn. If $m\angle ABC = 20$, find $m\overset{\frown}{BD}$.




$m\overline{BD} = 40°$

As shown in the diagram below, quadrilateral $DEFG$ is inscribed in a circle and $m\angle D = 86$.

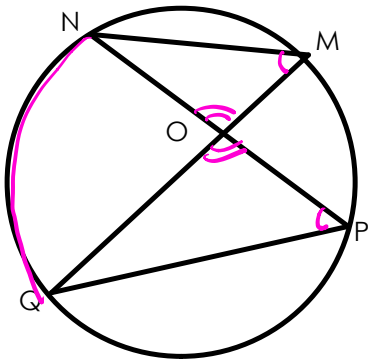



Determine and state $m\overset{\frown}{GFE}$. Determine and state $m\angle F$.

$m\overset{\frown}{GFE} = 172°$

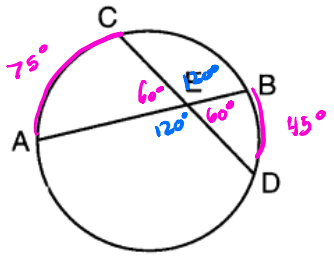
$m\angle F = 94°$

Prove △MON ~ △PQO




| Statement | Reason |
|---|---|
| ① $\angle NMQ \cong \angle NPQ$ | ① inscribed ∠'s that intercept the same arc are ≅ |
| ② $\angle NOM \cong \angle QOP$ | ② Vertical ∠'s are ≅ |
| ③ $\triangle MON \sim \triangle PQO$ | ③ AA ~ |

In the accompanying diagram, chords $\overline{AB}$ and $\overline{CD}$ intersect at $E$. If $m\overset{\frown}{AC} = 75$ and $m\overset{\frown}{DB} = 45$, find $m\angle AED$.



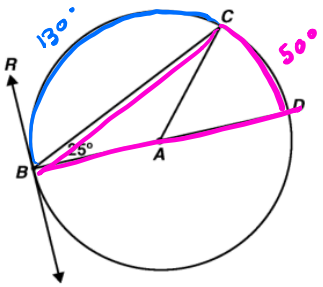

$\angle = \frac{Arc_1 + Arc_2}{2}$

$\angle = \frac{75 + 45}{2}$

$\angle = 60°$

$\angle AED = 120°$ (Linear pairs)

$\overline{RB}$ is tangent to a circle, whose center is $A$, at point $B$. $\overline{BD}$ is a diameter.



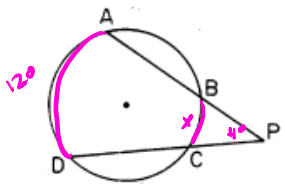

What is $m\angle CBR$?

$\angle = \frac{Arc}{2}$

$\angle = \frac{130}{2}$

$\angle CBR = 65°$

In the accompanying diagram, $\overline{PBA}$ and $\overline{PCD}$ are secants to the circle. If $m\angle P = 40$ and $m\overset{\frown}{AD} = 120$, find $m\overset{\frown}{BC}$.




$\angle = \frac{Big\ Arc - Small\ Arc}{2}$

$2\left[40 = \frac{120 - x}{2}\right]2$

$80 = 120 - x$

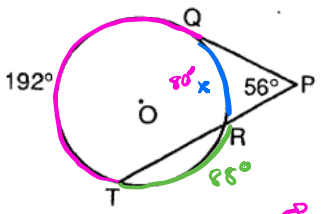
$-120 \quad -120$

$-40 = -x$

$40 = x$

$m\overset{\frown}{BC} = 40°$

In the accompanying diagram, $\overline{PQ}$ is tangent to circle $O$ at $Q$ and $\overline{PRT}$ is a secant. If $m\angle P = 56$ and $m\overset{\frown}{QT} = 192$, find $m\overset{\frown}{TR}$.




$\angle = \frac{Big\ Arc - Small\ Arc}{2}$

$2\left[56 = \frac{192° - x}{2}\right]2$

$112° = 192° - x$

$-192 \quad -192$

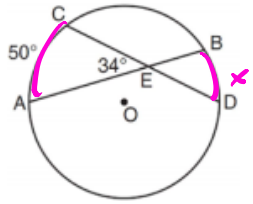
$-80 = -x$

$80° = x$

$\overset{\frown}{TR} = 360 - 192 - 80 - 88$

$\overset{\frown}{TR} = 88°$

In the diagram below of circle $O$, chords $\overline{AB}$ and $\overline{CD}$ intersect at $E$.




If $m\angle AEC = 34$ and $m\overset{\frown}{AC} = 50$, what is $m\overset{\frown}{DB}$?

$\angle = \frac{Arc_1 + Arc_2}{2}$

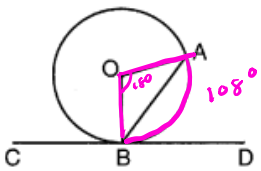
$2\left[34° = \frac{50 + x}{2}\right]2$

$68 = 50 + x$

$-50 \quad -50$

$18° = x$

In the accompanying diagram, $\overline{CD}$ is tangent to circle $O$ at $B$, $\overline{AO}$ and $\overline{BO}$ are radii, and chord $\overline{AB}$ is drawn. If $m\angle AOB = 108$, find $m\angle ABD$.




$\angle = \frac{Arc}{2}$

$\angle = \frac{108}{2}$

$\angle = 54°$

## Formulas You Need To Know

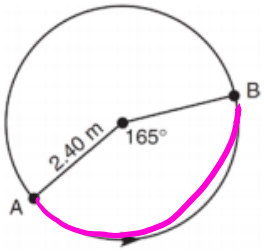
Formula: Arc Length

$$l = \frac{\theta}{360} 2\pi r$$

Formula: Area of a Sector

$$Area = \frac{\theta}{360} \times \pi r^2$$

The accompanying diagram shows the path of a cart traveling on a circular track of radius 2.40 meters. The cart starts at point $A$ and stops at point $B$, moving in a counterclockwise direction. What is the length of minor arc $AB$, over which the cart traveled, to the *nearest tenth of a meter*?

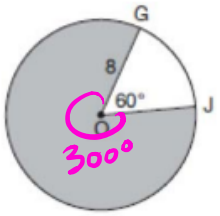



$l = \frac{\theta}{360} \cdot 2\pi r$

$l = \frac{165}{360} \cdot 2\pi(2.4)$

$l = 6.9$

In the diagram below of circle $O$, $GO = 8$ and $m\angle GOJ = 60°$.




What is the area, in terms of $\pi$, of the shaded region?

$A = \frac{\theta}{360} \cdot \pi r^2$

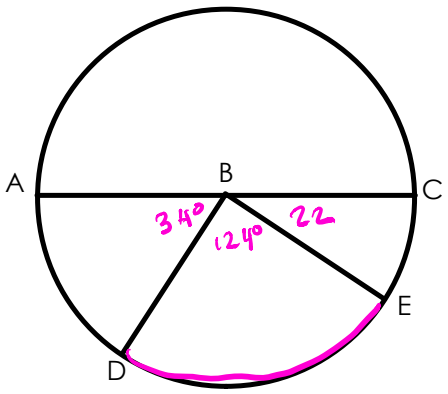
$A = \frac{300}{360} \cdot \pi 8^2$

$A = \frac{5}{6} \cdot \frac{64}{1} \cdot \pi$

$A = \frac{320}{6}\pi$

$A = \frac{160}{3}\pi$

The diameter of of circle B is $\overline{AC}$, which has a length of 20cm. The $m\angle ABD = 34°$ and the $m\angle CBE = 22°$. Find the length of $\overset{\frown}{DE}$.

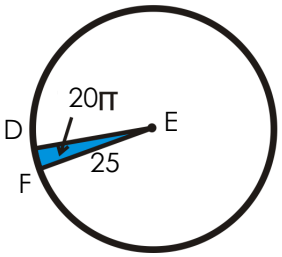



$l = \frac{\theta}{360} \cdot 2\pi r$

$l = \frac{124}{360} \cdot 2\pi \cdot 10$

$l = 21.64$

In the diagram below of circle E, the area of the shaded sector DEF is $20\pi$ and the length of the radius is 25. Determine the $m\angle E$




$A = \frac{\theta}{360} \cdot \pi r^2$

$20\pi = \frac{x}{360} \cdot \pi (25)^2$ (multiply by 360)

$7200 = 625x$

$11.52 = x$

$m\angle E = 11.52°$

Find the circumference to the nearest tenth of a circle with the points (4, 6) and (–2, 9) as the endpoints of a diameter.

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$d = \sqrt{(4 - (-2))^2 + (6 - 9)^2}$

$d = \sqrt{6^2 + (-3)^2}$

$d = \sqrt{36 + 9}$

$d = \sqrt{45}$

$d = 6.7$

$r = 3.35$

$C = 2\pi r$

$C = 2\pi(3.35)$

$C \approx 9.6$

Which point is on the circle whose equation is $x^2 + y^2 = 289$?

1) (–12, 12)
2) (7, –10)
3) (–1, –16)
4) (8, –15)

$(8)^2 + (-15)^2 = 289$

$64 + 225 = 289$

$289 = 289$ ✓

Find the coordinates of the center and the radius of the circle with diameter $\overline{CD}$. $C(-6, 2)$, $D(8, 6)$

[A] (1, 4), $\sqrt{55}$ [B] (–7, –2), $\sqrt{15}$

[C] (–7, –2), $\sqrt{17}$ [D] (1, 4), $\sqrt{53}$

What is the equation of this circle?

$M = \frac{-6+8}{2}, \frac{2+6}{2}$

$M = \frac{2}{2}, \frac{8}{2}$

$M = 1, 4$

C: 1, 1

r: $\sqrt{53}$

$(x-1)^2 + (y-4)^2 = 53$

A circle whose center is the origin passes through the point (–5, 12). Which point also lies on this circle?

1) (10, 3)
2) (–12, 13)
3) $(11, 2\sqrt{12})$
4) $(-8, 5\sqrt{21})$

$d = \sqrt{(-5-0)^2 + (12-0)^2}$

$d = \sqrt{25 + 144}$

$d = \sqrt{169}$

$d = 13$

$x^2 + y^2 = 169$

$11^2 + (2\sqrt{12})^2 = 169$

$121 + 48 = 169$

$169 = 169$ ✓

Find the radius and the center of the circle with the equation below

$x^2 + y^2 + 14y + 10x - 7 = 0$

$(x+5)^2 + (y+7)^2 = 81$

C: (–5, –7)

r: 9

Find the center and the radius from the equation of the circle below

$x^2 + y^2 + 2x - 30y + 22 = 0$

$(x+1)^2 + (y-15)^2 = 204$

C: (–1, 15)

R: $\sqrt{204}$